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## Observability of quantum state of black hole

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### ABSTRACT

We analyze terms subleading to Rutherford in the  $S$ -matrix between black hole and probes of successively high energies. We show that by an appropriate choice of the probe one can read off the quantum state of the black hole from the  $S$ -matrix, staying asymptotically far from the BH all the time. We interpret the scattering experiment as scattering off classical stringy backgrounds which explicitly depend on the internal quantum numbers of the black hole.

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## 0 Introduction

One of the most mysterious aspects of black hole physics is the no-hair theorem. In its classic form it appears to declare unobservable any attribute other than the mass  $M$ , charge  $Q$  and angular momentum  $J$  of a black hole [1]. The mystery deepens with Hawking’s discovery that even after quantum effects are switched on, the radiation emitted is apparently thermal, with a temperature  $T(M, Q, J)$ , thus limiting the observable information once again to those three quantities. In the context of a gravitational collapse, this would appear to suggest (to the radically-inclined) that the process of collapse reduces any arbitrary configuration of collapsing matter to a *unique* quantum state characterized by the above three quantities. While the violation of various global conservation laws implied by such a scenario is not entirely unthinkable, many distinct initial states evolving to the same final state is inconsistent with unitarity: a basic tenet of quantum mechanics. Furthermore, it rules out any thermodynamic understanding of the Hawking-Beckenstein entropy as the logarithm of the number of states.

One possible way out of this would be to say that there is no unique final state, but so far as the external world is concerned, all those states are *indistinguishable*. Although such a position avoids the two objections mentioned in the last paragraph, it seems to allow, in a sense, unobservable *observables*.<sup>1</sup>

In this paper we study the issue of *observability* of the “internal” states of the black holes in the context of black hole models in string theory. We begin the discussion by considering the electrically charged black holes of [4](Sec. 1). The states here are given explicitly by conformal field theory vertex operators. Various states differ in the choice of internal polarizations in the compact directions. In [2] the problem of scattering of probes off such black holes was considered. In the limit of a black hole of large mass and low energy of probe (Rutherford limit) it was found that leading term in the scattering could be understood as scattering off a black hole metric which in particular did not depend on the internal polarization of the black hole. Already in [2] it was found that this no-hair property did not persist in higher order terms in the scattering matrix (beyond Rutherford) and that these showed

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<sup>1</sup>This is related to the more general issue of complementarity of descriptions of physical phenomena in the context of black holes.

dependence on the internal polarization tensors of the black hole. In this paper we take a more detailed look at the post-Rutherford terms. We find that they involve non-trivial entanglement between polarization tensors of the BH and polarization tensors (and charges) of the probe. Indeed, not only can we get *some* information about the internal polarization of the black hole from the S-matrix, by using appropriate probes, we can actually completely determine the state of the black hole. In other words, measurements from far away can uniquely determine the quantum state of the black hole. Thus, the black hole states are no different from ordinary string states in this regard. Recently Susskind et al [5] have argued that *any* (sufficiently massive) string state is a black hole after all. The observation that we have made above is consistent with this proposal.

The fact that the the scattering matrix contains more information about the quantum state of the black hole, beyond that allowed by the classical no-hair theorems, and the fact that the scattering matrix has the interpretation of scattering of probe particles off backgrounds of various string modes around the BH (Sec. 2), naturally leads us to ask the question: How do these backgrounds manage to carry the additional information? Larsen and Wilczek [6] have argued that the usual treatment of no-hair theorem is rather restrictive and that within a higher dimensional field theory Kaluza-Klein framework there are an infinite number of distinct classical solutions, corresponding to the same overall mass and charges, but differing in the background values of the Kaluza-Klein modes. We show that (Sec. 2) our scattering results imply that the elementary BPS state not only gives rise to backgrounds of metric, dilaton, etc. but also to backgrounds of an infinite number of higher string modes. Although the metric and dilaton backgrounds do not depend on the internal polarization tensor (at leading order), the massive string modes explicitly depend on them, e.g.

$$M_{ijkl}(\rho) = (1/m)e^{-\sqrt{2}\rho}/\rho [\zeta_{R,i}\zeta_{R,j} + Q_{R,i}Q_{R,j}][\zeta_{L,k}\zeta_{L,l} + 1/2Q_{L,k}Q_{L,l}] + o(g_{st}) \quad (0.1)$$

(see equation (2.10) in Sec. 2). It turns out that by measuring a sufficient number of these backgrounds we can actually determine the microstate of the black hole entirely. Inelastic amplitudes involving Kaluza-Klein charge exchange between the probe particles and the black hole also contain terms which show dependence on the internal polarization of the black hole, and, therefore, can be used to get information on the microstate of the black hole.

These amplitudes seem to have a close connection with the observations made in [6].

We believe that the scenario that we have presented above suggests that string theoretic models of black holes come packaged with an infinite number of higher mass string backgrounds (in addition to metric, dilaton and moduli) which contain information about the detailed state of the black hole. In Sec. 3 we make some remarks about hair on D-brane models of black holes. In Sec. 4 we present a summary and outlook.

## 1 Hair from $S$ -matrix

In this section we consider the electrically charged black holes of [4] and show that by scattering suitable probes off such a black hole it is possible to get information about (and, in fact, *determine*) its detailed quantum state.

We work in heterotic string compactified on  $T^6$ . Our notations are as follows (for details, see [2]). The bosonic coordinates are  $x^\mu$ ,  $\mu = 0, 1, 2, 3$ ,  $x_R^i$ ,  $i = 1, \dots, 6$ ,  $x_L^i$ ,  $i = 1, 2, \dots, 22$ .  $x_{R,L}^i$  are holomorphic and antiholomorphic respectively. World sheet fermions are  $\psi^\mu(z)$  and  $\psi_R^i(z)$ . Here  $R, L$  stand for right and left movers (analytic and antianalytic respectively in our convention). For a generic torus  $T^6$ , the gauge group is abelian:  $U(1)^{28}$ , arising from 6 right-moving and 22 left-moving currents. We will denote the corresponding charges as  $\vec{Q}_R$  and  $\vec{Q}_L$  resp. For BPS states the mass  $m$  satisfies the condition

$$m^2 = Q_R^2 = Q_L^2 + 2(N_L - 1) \quad (1.1)$$

where  $N_L$  is the oscillator level in the left-moving (antianalytic) sector. The black holes of [4] are BPS states represented by the vertex operators of the form

$$\begin{aligned} V_B(\zeta_R; \zeta_L; k; z, \bar{z}) &= V_B(\zeta_R, k, z) \bar{V}_B(\zeta_L, \bar{z}) \exp[iQ_R \cdot x_R + iQ_L \cdot x_L + ik \cdot x(z, \bar{z})] \\ V_B(\zeta, k, z) &= \zeta_R \cdot \psi_R(z) e^{-\phi(z)} \\ \bar{V}_B(\zeta_L, \bar{z}) &= \zeta_{L, i_1 i_2 \dots i_r} \partial_{\bar{z}}^{n_1} x_L^{i_1} \partial_{\bar{z}}^{n_2} x_L^{i_2} \dots \partial_{\bar{z}}^{n_r} x_L^{i_r} \end{aligned} \quad (1.2)$$

In the above  $k^2 = -m^2$  and

$$N_L = \sum_i n_i \quad (1.3)$$

Different black holes with the same mass  $m$  and charges  $\vec{Q}_{R,L}$  differ in the choice of the internal polarization tensor  $\zeta_L$  (they differ in  $\zeta_R$  also, but for large  $N_L$  the main degeneracy comes from varying  $\zeta_L$ 's). For simplicity we have here chosen the polarization tensors  $\zeta_{L,R}$  entirely in the compact directions.

In the following we will use various probes to extract information about the state of the black hole, or in other words about the polarization  $\zeta_L$  (and  $\zeta_R$ ). We discuss the various choices of probes in turn:

(a) Massless probes:

These are given by vertex operators

$$\begin{aligned} \mathbf{V}_P(\eta_R; \eta_L; k; z, \bar{z}) &= V_P(\eta_R, k, z) \bar{V}_P(\eta_L, \bar{z}) \exp[ik \cdot x(z, \bar{z})] \\ V_P(\eta_R, k, z) &= \eta_{R,M} (\partial_z x^M + ik_\mu \psi^\mu \psi^M) \\ \bar{V}_P(\eta_L, \bar{z}) &= \eta_{L,N} \partial_{\bar{z}} x^N \end{aligned} \quad (1.4)$$

The case when the polarization vectors  $\eta_{R,L}$  have components only in the compact directions corresponds to the moduli fields as probe. The four point amplitude [2] describing the scattering of these probes off the black hole is as follows.

#### $N_L = 1$ black holes

For  $N_L = 1$  black holes the amplitude is

$$\begin{aligned} \mathcal{M}(1, 2, 3, 4) &= A_1(s, t, u) \times \\ &(\zeta_R \cdot \zeta'_R \eta_R \cdot \eta'_R + [\frac{t}{s-m^2} \zeta_R \cdot \eta_R \zeta'_R \cdot \eta'_R + \text{b.e.}] - \frac{2t}{(s-m^2)(u-m^2)} \eta_R \cdot \eta'_R \zeta \cdot Q_R \zeta' \cdot Q_R) \times \\ &(\zeta_L \cdot \zeta'_L \eta_L \cdot \eta'_L + [\frac{(t+2)t}{(2+s-m^2)(s-m^2)} \zeta_L \cdot \eta_L \zeta'_L \cdot \eta'_L + \text{b.e.}] - \frac{t(t+2)}{(s-m^2)(u-m^2)} \eta_L \cdot \eta'_L \zeta \cdot Q_L \zeta' \cdot Q_L), \\ A_1(s, t, u) &= -\pi \Gamma(-\frac{t}{2}) \Gamma(\frac{m^2-u}{2} + 1) \Gamma(\frac{m^2-s}{2} + 1) / [\Gamma(\frac{t}{2} + 2) \Gamma(\frac{u-m^2}{2}) \Gamma(\frac{s-m^2}{2})] \end{aligned} \quad (1.5)$$

Here b.e. represent Bose exchange of particles 2 and 4. <sup>2</sup>

### Determination of BH polarizations:

The various terms in (1.5) involve *different* functions of the Mandelstam variables and are hence independently measurable from experiments. Of importance to us is the (post-Rutherford) term

$$\begin{aligned}\mathcal{M}(1, 2, 3, 4) &= \dots + \zeta_R \cdot \eta_R \zeta'_R \cdot \eta'_R \zeta_L \cdot \eta_L \zeta'_L \cdot \eta'_L f(s, t, u) + \dots, \\ f(s, t, u) &\equiv \frac{(t/2)^2(t/2+1)}{[(s-m^2)/2]^2[(s-m^2)/2+1]} A_1(s, t, u) = \sum_{n=0}^{\infty} \frac{f_n(s)}{t-(2n+2)}, \\ f_0(s) &= -2\pi(1 + (s - m^2)/2)\end{aligned}\tag{1.6}$$

An experimental measurement of this term amounts to a measurement of the combination  $\zeta_R \cdot \eta_R \zeta'_R \cdot \eta'_R \zeta_L \cdot \eta_L \zeta'_L \cdot \eta'_L f(s, t, u)$ . It is therefore easy to see that by repeating the experiment with various choices of  $\eta_{L,R}$  we can determine the initial polarizations  $\zeta_{L,R}$  of the black hole. The fact that  $f(s, t, u)$  has no pole at  $t = 0$  implies that the information about the polarizations  $\zeta$  is propagated by stringy modes. We will have more to say on this in the next section on stringy classical backgrounds.

### Black holes with $N_L > 1$ :

The calculation for  $N_L = 1$  can be generalized in a straightforward fashion to higher  $N_L$ . In this case there are higher tensors  $\zeta_L$  which describe polarization of the black hole, e.g.  $\zeta_{L,i_1 i_2 \dots i_n}$ . Once again there are post-Rutherford terms, similar to (1.7). The general term involving entanglement

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<sup>2</sup>Our general notation for amplitudes is as follows. In the four-point scattering amplitude  $\mathcal{M}(1, 2, 3, 4)$  particles 1,3 refer to incoming and outgoing BH states and 2,4 refer to incoming and outgoing states of the probe respectively. The polarization tensors of the outgoing states are denoted by primes: thus the incoming polarizations of BH's are  $\zeta_{L,R}$  and outgoing polarizations are  $\zeta'_{L,R}$ . Similarly those for the probes are  $\eta_{L,R}$  and  $\eta'_{L,R}$  respectively. The charges of the particles 1,2,3,4 (when they are non-zero) will be denoted by  $Q_{L,R}, q_{L,R}, Q'_{L,R}$  and  $q'_{L,R}$  respectively. All momenta and charges will be taken as incoming so that momentum conservation reads as  $\sum_{i=1}^4 k_{i,\mu} = 0$  and charge conservation reads as  $Q_L + Q'_L + q_L + q'_L = Q_R + Q'_R + q_R + q'_R = 0$ . The Mandelstam variables are defined as usual:  $s = -(k_1 + k_2)^2, t = -(k_1 + k_3)^2, u = -(k_1 + k_4)^2$ .

of polarizations of BH and probe is of the form

$$\begin{aligned} \mathcal{M}(1, 2, 3, 4) &= \dots + \zeta_R \cdot \eta_R \zeta'_R \cdot \eta'_R \eta_L^i \zeta_{L, i j_1 j_2 \dots j_n} \zeta_L^{i k j_1 j_2 \dots j_n} \eta'_{L, k} g(s, t, u) + \dots, \\ g(s, t, u) &= \sum_{n=0}^{\infty} \frac{g_n(s)}{t - (2n+2)} \end{aligned} \quad (1.7)$$

Once again this term can be separated from the rest of the terms in the  $S$ -matrix by its momentum dependence. It is clear that by choosing the polarization of the massless probe appropriately, we can glean some information about the quantum state of the black hole. The information is, however, partial since most of the “legs” of the tensor  $\zeta_L$  do not contract with  $\eta_L$ . In order to improve this situation, we need to consider massive probes (neutral or charged).

(b) Massive probes:

Massive states can either be neutral or charged. A particle with charge  $q_{L,R}$  has a mass

$$m^2 = q_R^2 + 2(N_R - 1/2) = q_L^2 + 2(N_L - 1) \quad (1.8)$$

(i) Neutral probes:

Let us consider neutral probes first ( $q_{L,R} = 0, N_R - 1/2 = N_L - 1 = n$ ). Their masses are of the string scale:

$$m^2 = 2n \quad (1.9)$$

As in the case of the massless probes above (which are necessarily neutral), the charge of the black hole is unaffected by scattering with these probes:  $Q_{L,R} + Q'_{L,R} = 0$ , since  $q_{L,R} = q'_{L,R} = 0$ .

The new feature that arises in the 4-point  $S$ -matrix is that we start obtaining terms with increasing number of contractions between the polarizations of the probe and the black hole. For example, a massive probe with polarizations  $\eta_{R, i_0 i_1 \dots i_n}, \eta_{L, j_0 j_1 \dots j_n}$  has the following term in the  $S$ -matrix (we have considered BH vertex operators (1.2) with all  $n_i = 1$ ):

$$\begin{aligned} \mathcal{M}(1, 2, 3, 4) &= \dots + \zeta_R^{i_0} Q_R^{i_1} \dots Q_R^{i_n} \eta_{R, i_0 i_1 \dots i_n} \zeta'^{j_0} Q_R^{j_1} \dots Q_R^{j_n} \eta'_{R, j_0 j_1 \dots j_n} \\ &\quad \eta_L^{k_0 k_1 \dots k_n} \zeta_{L, k_0 k_1 \dots k_n p_1 p_2 \dots p_{N_L - n - 1}} \zeta_L^{l_0 l_1 \dots l_n p_1 p_2 \dots p_{N_L - n - 1}} \eta'_{L, l_0 l_1 \dots l_n} h(s, t, u) + \dots, \\ h(s, t, u) &= \sum_{m=0}^{\infty} h_m(s) / (t - (2m + 4n + 2)) \end{aligned} \quad (1.10)$$

A measurement of this term for various choices of the probe polarizations  $\eta_{L,R}$  ultimately *determines* for us  $\zeta_{L,R}$  in case  $n = N_L$ , that is, when the (left) oscillator level of the probe matches the (left) oscillator level of the black hole. In the next section (on classical backgrounds) we will see that this corresponds to a  $\zeta$ -dependent non-zero background value of a string mode ( $\text{mass}^2 = 2n$ ) around the black hole. We should note here that such a background can be detected either by a single massive probe (as described by a four-point function) or by many massless probes (which involves an  $n$ -point amplitude for large  $n$ ).

(ii) Charged Probes:

(1) Kaluza-Klein probes:

Let us first consider probes whose 10-dimensional masses vanish, in other words, probes with  $N_R - 1/2 = N_L - 1 = 0$ . Their (four-dimensional) masses are

$$m^2 = \mathbf{q}_R^2 = \mathbf{q}_L^2 \quad (1.11)$$

which are of the order of the compactification scale. We shall call these probes KK probes.

With charged probes we can have two kinds of amplitudes, one in which the charge does not change (neutral channel) and another in which the charge changes (charged channel):

Neutral channel

This is the case in which the charge of the probe does not change:  $\mathbf{q}_{L,R} = -\mathbf{q}'_{L,R}$ . The charge of the black hole also remains the same. Vis-a-vis the polarization tensors  $\zeta$ , the  $S$ -matrix in this case does not give any more information than in the massless case discussed above.

Charged channels

These are amplitudes in which the charge of the black hole and the charge of the probe are allowed to change:

$$Q_{L,R} + Q'_{L,R} = \mathbf{q}_{L,R} + \mathbf{q}'_{L,R} = \Delta\mathbf{q}_{L,R} \neq 0 \quad (1.12)$$

This also typically implies a change of  $N_L$ . We will call the final oscillator level of the black hole  $N'_L$ . Note that this is an inelastic process, that is, the



BH under measurement changes its state after the “measurement”. However, by measuring such amplitudes, we can still get useful information about the initial state of the black hole. Indeed,  $S$ -matrix elements of this type can completely determine the initial polarization tensor  $\zeta$  of the black hole. It can be shown that the  $S$ -matrix contains a term (which can be independently measured by choosing the momenta and charges of initial and final states of the probe appropriately)

$$\begin{aligned}
\mathcal{M}(1, 2, 3, 4) &= \dots + \zeta_R \cdot \eta_R \zeta'_R \cdot \eta'_R \\
&\Delta \mathbf{q}_{i_1} \Delta \mathbf{q}_{i_2} \dots \Delta \mathbf{q}_{i_{N_L}} \zeta_L^{i_1 i_2 \dots i_{N_L}} \Delta \mathbf{q}_{j_1} \Delta \mathbf{q}_{j_2} \dots \Delta \mathbf{q}_{j_{N'_L}} \zeta_L'^{j_1 j_2 \dots j_{N'_L}} F(s, t, u) \\
F(s, t, u) &= \sum_{n=0}^{\infty} \frac{F_n(s)}{t - (m_{KK}^2 + 2n)} \\
m_{KK}^2 &\equiv \Delta \mathbf{q}_R \cdot \Delta \mathbf{q}_R = \Delta \mathbf{q}_L \cdot \Delta \mathbf{q}_L
\end{aligned} \tag{1.13}$$

The second equality in the last line follows from the fact that we are considering exchange particles satisfying  $N_R - 1/2 = N_L - 1 = 0$ .

#### Determination of hair:

In the above  $\Delta \mathbf{q}_L \equiv \mathbf{q}_L - (-\mathbf{q}'_L)$  is the charge difference between the initial and final states of the probe and is therefore a known vector. By a sufficient number of experiments with various choices of  $\Delta \mathbf{q}_L$  we can ultimately determine the tensor  $\zeta_L$ . The determination of  $\zeta_R$  is trivial (by tailoring  $\eta_R$ ). Thus we are able to determine the initial state of the black hole by using Kaluza-Klein probes whose generation requires energies of the compactification scale rather than the string scale. Such a determination of the internal state of the black hole bears a close resemblance to the observations made in [6]. We will have more to say on this at the end of the next section.

(2) Charged probes of string mass: These do not contain any new physics and so we do not consider them here.

## 2 Hairy classical backgrounds

In this section we show how to interpret the earlier results as the scattering of probe particles off backgrounds of string modes, massless *as well as mas-*

sive, created by the BPS state. Let us denote the string mode in spacetime corresponding to the BPS state by  $\psi(x)$  (this field has indices:  $\psi^{i_1 i_2 \dots i_n}$  corresponding to polarization indices in the internal compact directions). The beta function equations for neutral fields, say  $\phi^{k_1 \dots k_m \mu_1 \dots \mu_l}$  are of the form [7, 8]

$$0 = \beta_\phi^{k_1 \dots k_m \mu_1 \dots \mu_l}(q) = (q^2 + m_\phi^2) \phi^{k_1 \dots k_m \mu_1 \dots \mu_l}(q) - g_{st} \int dk_1 dk_3 [\Gamma_\phi^{\psi\psi}]_{i_1 \dots i_n, j_1 \dots j_n}^{k_1 \dots k_m \mu_1 \dots \mu_l}(k_1, k_3) \psi^{* i_1 \dots i_n}(k_1) \psi^{j_1 \dots j_n}(k_3) \tilde{\delta}^4(q - k_1 - k_3) + \dots \quad (2.1)$$

which we will schematically write as (suppressing indices)

$$0 = (q^2 + m_\phi^2) \phi(q) - g_{st} \int dk_1 dk_3 \tilde{\delta}^4(q - k_1 - k_3) \Gamma_\phi^{\psi\psi}(k_1, k_3) \psi^*(k_1) \psi(k_3) + \dots \quad (2.2)$$

Here  $\Gamma_\phi^{\psi\psi}$  are determined by the operator product expansion coefficients [7, 8] of the vertex operators of the fields  $\psi, \psi$  and  $\phi$ . Also, we have used the notation  $dk \equiv d^4 k / (2\pi)^4$  and  $\tilde{\delta}^4(k) \equiv (2\pi)^4 \delta^4(k)$ .

We are interested in the BPS state, described by the field  $\psi$  (mass  $m$ ), to be in its rest frame and given by a wavefunction

$$\psi^{(0), i_1 \dots i_n}(k) = 2\pi \delta(k_0 - \omega_{\vec{k}}) \zeta^{i_1 \dots i_n} f(\vec{k}) / \sqrt{2\omega_{\vec{k}}}, \quad \omega_{\vec{k}}^2 \equiv \vec{k}^2 + m^2, \quad (2.3)$$

where  $f(\vec{k})$  is a wave-packet centered around  $\vec{k} = 0$  and satisfies  $\int \frac{d^3 \vec{k}}{(2\pi)^3} |f(\vec{k})|^2 = \int d^3 x \psi^* \overleftrightarrow{\partial}_0 \psi = 1$ .  $\zeta^{i_1 \dots i_n}$  denotes the (real) polarization tensor of the particle and satisfies the normalization condition  $\zeta^{i_1 \dots i_n} \zeta_{i_1 \dots i_n} = 1$ . The  $\phi$  background created by such a BPS state is obtained, to first order in  $g_{st}$ , by solving (2.2):

$$\phi^{(1)}(q) = g_{st} (q^2 + m_\phi^2)^{-1} \int dk_1 dk_3 \Gamma_\phi^{\psi\psi}(k_1, k_3, q) \psi^{*(0)}(k_1) \psi^{(0)}(k_3) \delta(k_1 + k_3 - q) \quad (2.4)$$

It is not difficult to show that both (2.3) and (2.4) are classical solutions of their respective equations of motion to first order in  $g_{st}$ . In the case of sufficiently peaked wave-packets  $f(\vec{k})$ , we can easily carry out the integral (2.4), after substituting for  $\psi^{(0)}$  from (2.3). We get

$$\phi^{(1), k_1 \dots k_m, \mu_1 \dots \mu_l}(q) = \frac{g_{st}}{2m} (q^2 + m_\phi^2)^{-1} [\Gamma_\phi^{\psi\psi}]_{i_1 \dots i_n, j_1 \dots j_n}^{k_1 \dots k_m, \mu_1 \dots \mu_l}(\vec{k}, \vec{k}, q) \zeta^{i_1 \dots i_n} \zeta^{j_1 \dots j_n} 2\pi \delta(q_0) \quad (2.5)$$

where  $\vec{k} \equiv (m, \vec{0})$  denotes the four-momentum of the BPS state in its rest frame.

In deriving the above solution we have used the condition of low momentum transfer ( $|\vec{q}| \ll m_{string}$ ). While we expect the solution to get modified at short distances due to string world sheet corrections [4], it should remain valid at large distances. We should also mention that (2.5), together with (2.3), represent classical backgrounds corresponding to an *infinite* number of string modes, and it is the set of all these backgrounds which constitutes the string theory black hole.

#### Scattering off background:

Let us now find the scattering amplitude of a probe particle (represented by some string mode  $\chi(x)$  of mass  $\mu$  say) off the above background (2.5). The relevant part of the action is

$$S = \int dk (k^2 + \mu^2) \chi(k) \chi(-k) + g_{st} \int dk_2 dk_4 dq \Gamma_{\phi}^{\chi\chi} \chi(k_2) \chi(k_4) \phi(q) \delta(k_2 + k_4 + q) + \dots \quad (2.6)$$

The S-matrix describing the amplitude for scattering of the field  $\chi(x)$  ( $k_2, k_4$  are the initial and final momenta and  $\eta, \eta'$  are the initial and final polarizations) is given by

$$\mathcal{S} = \frac{\phi^{(1)} \Gamma_{\phi}^{\chi\chi} \eta \eta'}{\sqrt{2k_2^0} \sqrt{2k_4^0}} \quad (2.7)$$

where we have dropped the momentum conserving delta-functions. If we substitute for  $\phi^{(1)}$  from (2.4), we get

$$\mathcal{S} = \frac{\Gamma_{\phi}^{\psi\psi} \zeta \Gamma_{\phi}^{\chi\chi} \eta \eta'}{(k_2 + k_4)^2 + m_{\phi}^2} \frac{1}{\sqrt{2k_1^0} \sqrt{2k_2^0} \sqrt{2k_3^0} \sqrt{2k_4^0}} \quad (2.8)$$

where  $k_1^0 = k_3^0 = m$ .

#### Comparison with 4-point amplitude:

It is easy to show that (2.8) is precisely the  $S$ -matrix for the process in which the particle  $\chi(x)$  with initial momentum  $k_2$  and initial polarization  $\eta$  is scattered to a state  $(k_4, \eta')$  because of a  $\phi$ -particle exchange with a static

BPS state with mass  $m$  and polarization  $\zeta$ . We see, therefore, that the 4-point  $S$ -matrices calculated in the previous section, factorized on specific channels, can be interpreted as scattering off classical backgrounds given by expressions like (2.5). The important thing to note is that these expressions explicitly involve the polarization tensor  $\zeta$  of the black hole.

### Determination of the backgrounds

The strategy for determining the (first order) backgrounds around the BPS state is now clear. We either use (2.5) directly, or solve (2.7) for  $\phi^{(1)}$  from a knowledge of  $\mathcal{S}$ , the 4-point  $S$ -matrix. As we have shown above, the two procedures are equivalent.

We now list some backgrounds which we determine using this method. For massless exchanges, we get [9]

$$h_{\mu\nu}^{(1)}(\rho) = m/\rho\delta_{\mu 0}\delta_{\nu 0}, \quad \Phi^{(1)}(\rho) = -m/(2\rho) \quad (2.9)$$

These agree with the first order backgrounds of metric and dilaton as given in [4]. For the metric this agreement has already been shown in [2].

Massive backgrounds:

The amplitude (1.6) does not have a massless  $t$  channel. The lightest exchange particles are of mass  $m^2 = 2$ . We list below the background value of one of these, corresponding to vertex operator  $V^{ij}\bar{\partial}x^k\bar{\partial}x^l$ <sup>3</sup> (again determined using the method outlined above)

$$M_{ijkl}^{(1)}(\rho) = (1/m)e^{-\sqrt{2}\rho}/\rho [\zeta_{R,i}\zeta_{R,j} + Q_{R,i}Q_{R,j}][\zeta_{L,k}\zeta_{L,l} + 1/2Q_{L,k}Q_{L,l}] + o(g_{st}) \quad (2.10)$$

Thus we have backgrounds which explicitly carry hair.

In case of higher  $N_L$  black holes, the above string mode has a background value (corresponding to (1.7))

$$M_{ijkl}^{(1)}(\rho) = (1/m)e^{-\sqrt{2}\rho}/\rho[\zeta_{R,i}\zeta_{R,j}\zeta_{L,kj_1\dots j_n}\zeta_{L,l}^{j_1\dots j_n} + \dots] + o(g_{st}) \quad (2.11)$$

In the above, the ellipsis represents additional terms which depend on the polarization and the charge vectors. These terms appear in the  $S$ -matrix elements separately. Clearly, because of the internal contractions within the  $\zeta$ 's these backgrounds do not carry enough hair to determine the  $\zeta$ 's completely.

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<sup>3</sup> $V^{ij} = \partial x^i\partial x^j + \partial\psi^i\psi^j + i(k.\psi)\psi^i\partial x^j$  is the supersymmetrized version of  $\partial x^i\partial x^j$ .

More massive backgrounds:

If we consider backgrounds of sufficiently heavy string modes, they carry enough “hair” so as to let us determine  $\zeta_L$  completely. In the context of four-point amplitudes these backgrounds are seen only by massive string modes. Thus the background corresponding to the lightest channel of the amplitude (1.10) is given by

$$\begin{aligned}
& M^{(1), i_0 i_1 \dots i_n, j_0 j_1 \dots j_n, k_0 k_1 \dots k_n, l_0 l_1 \dots l_n} \\
&= (1/m) [e^{-\sqrt{4n+2}} \rho / \rho] \zeta_R^{i_0} Q_R^{i_1} \dots Q_R^{i_n} \zeta_R^{j_0} Q_R^{j_1} \dots Q_R^{j_n} \\
& \quad \zeta_{L, p_1 p_2 \dots p_{N_L-n-1}}^{k_0 k_1 \dots k_n} \zeta_L^{l_0 l_1 \dots l_n p_1 p_2 \dots p_{N_L-n-1}}
\end{aligned} \tag{2.12}$$

Clearly for  $n$  sufficiently large, there will be no internal contractions between the  $\zeta$ ’s and therefore by measuring sufficiently heavy backgrounds we can see enough “hair” so as to determine the polarization tensors  $\zeta$ .

#### Interpretation of scattering involving charged channels:

The background fields determined by the above procedure correspond to neutral excitations of the string. As we have seen, however, (in the subsection containing equations (1.12) and (1.13)), inelastic amplitudes corresponding to a charge exchange between the BH and the probe particles, are also useful in determining the initial state of the black hole, and in fact, can be used to completely fix the initial state. The KK backgrounds (hair) in the classical solution in [6] seem to correspond to such charge exchanges between the BH and the probe particles. In this connection we should note that [10] identifies the collective string Hilbert space, obtained by quantizing the functions  $\vec{f}_a(u), p_a(u)$  and  $q_{L,a}^I(u)$  (see Sec. 2.2 of [10]) appearing in the classical solution, to be the Hilbert space of the elementary string.

We can, in fact, make a more precise connection of the scattering amplitudes involving charged channels with the long-range KK ‘hair’ emphasized in [6]. At a first sight it would seem that the scattering in the charged channels is suppressed, at low momentum transfer, by a factor of  $1/m_{KK}^2$  because of the propagator of the KK particle,  $1/(t - m_{KK}^2)$ . However, if we consider processes in which the final state of the black hole is also a BPS state, with a charge vector  $Q_R$  which is collinear (but not identical) with  $Q'_R$ <sup>4</sup>, then, at low

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<sup>4</sup>We note that the KK backgrounds considered in [6] correspond to momentum in the

momentum transfer, the kinematics of the process ‘conspires’ to remove the  $1/m_{KK}^2$  suppression! To see this, note that the 4-momentum carried by the exchanged particle (in the rest frame of the initial BPS state) is  $q_\mu = (q_0, \vec{q})$  where

$$q_0 = \sqrt{m'^2 + |\vec{q}|^2} - m, \quad \vec{q} = -(\vec{k}_1 + \vec{k}_3), \quad (2.13)$$

where  $m = |Q_R|$  and  $m' = |Q'_R|$  are the masses of the initial and the final BPS state. In the limit  $|\vec{q}| \ll m'$ , we have  $q_0 \approx m' - m$ . Now, the mass of the exchanged KK particle is given by  $m_{KK}^2 = |Q'_R - Q_R|^2$ . For collinear charges  $Q'_R$  and  $Q_R$ , this is exactly equal to  $(m' - m)^2$ . We, therefore, find that for such processes the propagator for the exchanged KK particle goes as

$$\frac{1}{t - m_{KK}^2} = -\frac{1}{|\vec{q}|^2} \quad (2.14)$$

which is characteristic of a long-range interaction! It is important to note here that for such processes the term in the charge exchange amplitude given in (1.13) is non-zero only for those black holes which carry charges in the sixteen toroidal directions of the uncompactified heterotic string and have polarizations in these directions as well as in the KK directions.

The charge exchange scattering processes identified above are obviously not generic enough to determine the internal polarization tensor of the black hole completely. To do this, we need to consider the more general processes in which  $Q_R$  and  $Q'_R$  are not collinear. In this case, the ‘miracle’ in (2.14) does not happen, and we have a suppression of these amplitudes by  $1/m_{KK}^2$ . Nevertheless, it is interesting that the mass scale at which information about the initial polarization tensor of the black hole can be obtained from scattering experiments is  $m_{KK}$  and not  $m_{string}$ . Such a conclusion has been anticipated in [6, 10].

### 3 D-branes

So far we have discussed the case of electrically charged black holes. It would clearly be interesting to see how the above ideas apply to the case of fat black

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$x^9$  direction, hence the charge vectors are collinear in an obvious sense; similarly our requirement of the final state satisfying the BPS condition is similar to their requirement that the background be supersymmetric.

holes described by D-branes [12]. This work is in progress, but we have some preliminary results for single D-branes carrying single open string excitations. We will show below that amplitudes involving the scattering of closed strings off such excited D-branes encode information about the polarization of the open string excitations.

We consider type II superstring in  $R^{d-1,1} \times T^p$ ,  $d + p = 10$  and a  $Dp$ -brane [13] wrapped on the  $T^p$ . Our notation for the spacetime coordinates is  $x^M = (x^\mu, x^i)$ ,  $\mu = 0, 1, \dots, d-1$ ;  $i = 1, \dots, p$ . We will also use the notation  $x^a = (x^0, x^i)$  for directions parallel to the D-brane, which include time. We consider a single open string excitation on the  $D$ -brane with polarization  $\zeta^a = (\zeta^0, \zeta^i)$  which is parallel to the D-brane. The vertex operator of this excitation is

$$V_E(\zeta^a, p^a, z) = \zeta_a(\partial x^a + ip_b \psi^b \psi^a) \exp(ip \cdot x(z)) \quad (3.1)$$

Note that this excitation moves along the compactified directions, so its momenta  $p^i$  are quantized. We now consider scattering the following closed string probes off the excited D-brane:

$$\begin{aligned} V_P(\eta_R, \eta_L; k^\mu; \mathbf{q}^i; z, \bar{z}) = \\ V_P(\eta_R, k, \mathbf{q}, z) \bar{V}_P(\eta_L, k, \mathbf{q}, \bar{z}) \exp[ik_\mu x^\mu(z, \bar{z}) + i\mathbf{q}_i x^i(\bar{z})], \\ V_P(\eta_R, k, \mathbf{q}, z) = \eta_{R,M}(\partial x^M + (ik_\mu \psi^\mu(z) + i\mathbf{q}_i \psi^i(z))\psi^M(z)), \\ \bar{V}_P(\eta_L, k, \mathbf{q}, \bar{z}) = \eta_{L,M}(\bar{\partial} x^M + (ik_\mu \psi^\mu(\bar{z}) + i\mathbf{q}_i \psi^i(\bar{z}))\psi^M(\bar{z})) \end{aligned} \quad (3.2)$$

where  $k$  is the space time momentum and  $\mathbf{q}$  is the KK charge. We consider for simplicity probes with no winding modes:  $\mathbf{q}_R = \mathbf{q}_L$ ; these turn out to be sufficient for our purposes here. The connected amplitude for the process at tree level is a disc diagram with two open strings at the boundary and two closed strings in the interior. It is easy to compute the part of this amplitude which arises from the exchange of closed strings which are massless in the ten-dimensional sense. One such term in the  $S$ -matrix (which can be separately measured by choosing the probe momenta and charges appropriately)

is reproduced below

$$\begin{aligned}
\mathcal{M}(1, 2, 3, 4) = & \dots + [16\pi q_{\parallel}^4 \Gamma(2q_{\parallel}^2) / (\Gamma(1 + q_{\parallel}^2))^2 (q^2 + m_{KK}^2)] \times \\
& \left( \zeta' \cdot \eta_R (\eta_{L,\mu} q^\mu + \eta_{L,i} \Delta \mathbf{q}^i) (\eta'_{R,\mu} q^\mu + \eta'_{R,i} \Delta \mathbf{q}^i) \zeta \cdot \eta'_L + \text{b.e.} \right) \times \\
& \left( \zeta' \cdot \eta_L (\eta_{R,\mu} q^\mu + \eta_{R,i} \Delta \mathbf{q}^i) (\eta'_{L,\mu} q^\mu + \eta'_{L,i} \Delta \mathbf{q}^i) \zeta \cdot \eta'_R + \text{b.e.} \right) + \dots
\end{aligned} \tag{3.3}$$

In the above  $q = k_2 + k_4$ ,  $q_{\parallel} = -q_0^2 + \vec{q}^2$  and  $\Delta \mathbf{q} (\equiv \mathbf{q} + \mathbf{q}')$  denotes the charge difference between the initial and final states of the probe. It is clear that by choosing probes appropriately we can determine the polarization  $\zeta^a$ .

We should note that the above amplitude corresponds to the exchange of a charged closed string (KK particle). It can be shown that neutral massless channels do not exhibit any ‘hair’, that is, they do not have any entanglement between the polarizations of the probe and the open string excitation on the D-brane.

#### Hair in absorption and decay amplitudes of fat black holes:

The above calculation addresses a BPS black hole made up of only a single D-brane and open string excitations on it. The construction of fat black holes [12], of course, involves multiple D-branes and open string excitations between them. It is an interesting question in that case whether one needs open string probes<sup>5</sup> to completely determine the states of a black hole, since the latter form a non-trivial representation of a  $U(N)$  gauge theory where closed string probes are singlets under such gauge groups. Work on this problem is in progress and we hope to come back to this question. Meanwhile, we close this section with a brief remark about an  $S$ -matrix calculation presented in [3] which exposes more detail about the quantum state of a fat black hole than is warranted by the classical no-hair theorems. These black holes are represented by left- and right-moving open string excitations on D-branes (for more detail, we refer the reader to [3] and references therein) and their detailed quantum state is represented by eqn. (6) of [3] which involves the number distribution of open strings with specific left- and right-moving momenta. The  $S$ -matrix element involved in the absorption of a closed string quantum by the black hole (same as the  $S$ -matrix for the decay of closed strings from the black hole) is given by eqn. (8) of [3]. If the closed string

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<sup>5</sup>We thank F. Larsen and F. Wilczek for raising this issue.



quantum coming out in the decay process has an energy  $\omega$  integer, then the  $S$ -matrix actually involves the number of left- and right-moving open strings with momentum  $\omega/2$  (denoted as  $N_{L,R}(m)$  in eqn. (8) of [3]) and the latter can therefore be measured from  $S$ -matrix data. Note that it is only the microcanonical average of these number distributions (and not the individual number for each state) which is determined by the temperature of the black hole. The individual distributions of left- and right-movers obviously have much more information than is contained in the data allowed by no-hair theorems.

## 4 Conclusion

In this paper we have presented a computation of  $S$ -matrix for scattering of probe particles off a black hole and shown how a measurement of the post-Rutherford terms can uniquely determine the microstate of the black hole from the  $S$ -matrix data. The calculation has primarily been carried out for the electrically charged black holes of [4] and some preliminary results have been presented for D-branes. The fact that we are able to determine the state of the black hole from an  $S$ -matrix is consistent with the identification of the black hole state with an elementary string state; however, it immediately raises the interesting question of what happened to the usual no-hair theorems of general relativity, which seem to preclude such detailed measurement of the state of the black hole from outside. This question becomes particularly intriguing in the light of our observation (Sec. 2) that some of the measurements only require energies of the order of the compactification scale which can be far less than the string scale. Our statements about the  $S$ -matrix appear to be closely connected to the observations made in [6] about violation of no-hair theorems in the context of classical solutions. Indeed, we explicitly demonstrate the interpretation of some of our  $S$ -matrix elements in terms of stringy classical backgrounds which carry information about the detailed state of the black hole. We would like to remark that in regimes where differential equations satisfied by various string modes cannot be trusted, the  $S$ -matrix may provide a more operational *definition* of various backgrounds (this is similar to the operational definition of horizon area proposed in [5] as the absorption cross-section). Such an  $S$ -matrix approach

to a consistent unitary quantum mechanics for black holes has actually been advocated as a principle by 't Hooft [15]. According to such a philosophy, it is the emergence of the classical no-hair theorems from a unitary  $S$ -matrix which requires an explanation. What we have seen in the present work is that the terms in the amplitude that exhibit the no-hair property are associated with massless exchanges; in the presence of string modes or Kaluza-Klein modes this property is lost, thus enabling determination of the state of a black hole, much like in the case of ordinary matter.

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## References

- [1] B. Carter, Phys. Rev. Lett. 26 (1971) 331; in *General Relativity, an Einstein centenary survey*, ed. S. W. Hawking and W. Israel (Cambridge, 1979)  
R.H. Price, Phys. Rev. D5 (1972) 2419, 2439  
K. Thorne, *Black Holes and Time Wraps: Einstein's Outrageous Legacy*, (W.W. Norton and Co., 1994)
- [2] G. Mandal and S.R. Wadia, Phys. Lett. B (1996) 372 **hep-th 9511218**
- [3] A. Dhar, G. Mandal and S.R. Wadia, **hep-th 9605234**, to appear in Phys. Lett. B
- [4] A. Sen, Mod. Phys. Lett. A10 (1995) 2081 **hep-th 9504147**
- [5] E. Halyo, B. Kol, A. Rajaraman and L. Susskind, **hep-th 9609075**
- [6] F. Larsen and F. Wilczek, **hep-th 9604134 and 9609084**
- [7] A.M. Polyakov, Physica Scripta T15 (1987) 191
- [8] I. Klebanov and L. Susskind, Phys. Lett. B200 (1988) 446

- [9] Similar methods have been used to determine background fields around D-branes in M.R. Garousi and R.C. Meyers, **hep-th 9603194**
- [10] C.G. Callan, J.M. Maldacena and A.W. Peet, **hep-th 9510134**
- [11] A. Dabholkar, J. Gauntlett, J. Harvey and D. Waldram, Nucl. Phys. B474 (1996) 85 **hep-th 9511053**
- [12] A. Strominger and C. Vafa, Phys. Lett. B379 (1996) 99 **hep-th 9601029**  
C. Callan and J. Maldacena, Nucl. Phys. B472 (1996) 591 **hep-th 9602043**  
J. Maldacena and L. Susskind, **hep-th 9604042**
- [13] J. Dai, R.G. Leigh and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073  
J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724 **hep-th 9510017**
- [14] A. Hashimoto and I. Klebanov, Phys. Lett. B381 (1996) 437 **hep-th 9604065**
- [15] G. 't Hooft, Int. J. Mod. Phys. A11 (1996) 4623 **gr-qc 9607022**